3. A test consisting of 25 multiple-choice questions with 5 answer choices for each question is administered. For each question, there is only 1 correct answer.

   (a) Let X be the number of correct answers if a student guesses randomly from the 5 choices for each of the 25 questions. What is the probability distribution of X?

   This test, like many multiple-choice tests, is scored using a penalty for guessing. The test score is determined by awarding 1 point for each question answered correctly, deducting 0.25 point for each question answered incorrectly, and ignoring any question that is omitted. That is, the test score is calculated using the following formula.

   Score $= (1 \times$ number of correct answers$) - (0.25 \times$ number of incorrect answers$) + (0 \times$ number of omits$)$

   For example, the score for a student who answers 17 questions correctly, answers 3 questions incorrectly, and omits 5 questions is

   $$\text{Score} = (1 \times 17) - (0.25 \times 3) + (0 \times 5) = 16.25.$$

   (b) Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. Show that the expected value of the student's score is 18 when using the scoring formula above.

   (c) A score of at least 20 is needed to pass the test. Suppose a student knows the correct answers for 18 questions, answers those 18 questions correctly, and chooses randomly from the 5 choices for each of the other 7 questions. What is the probability that the student will pass the test?

4. A company is considering implementing one of two quality control plans for monitoring the weights of automobile batteries that it manufactures. If the manufacturing process is working properly, the battery weights are approximately normally distributed with a specified mean and standard deviation.

Quality control plan A calls for rejecting a battery as defective if its weight falls more than 2 standard deviations below the specified mean.

Quality control plan B calls for rejecting a battery as defective if its weight falls more than 1.5 interquartile ranges below the lower quartile of the specified population.

Assume the manufacturing process is under control.

a. What proportion of batteries will be rejected by plan A ?

b. What is the probability that at least 1 of 2 randomly selected batteries will be rejected by plan A ?

c. What proportion of batteries will be rejected by plan B ?